Long-Range Service Life Analysis (LRSLA) Estimating Procedure

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Statistical techniques for estimating the service life of a solid rocket motor are applied once the computerized structural models have been validated by motor overtesting, dissection, and engineering analysis, for each mission-critical failure mode. Design and analysis of variance techniques are used to examine each structural model as a multiparameter (requirement) function dependent on the statistical behavior of "key" parameters with time. This method identifies and estimates the effects of the key parameters and approximates the structural models by linear functions of the key parameters and their standard deviations. One linear requirement function is probabilistically compared with the capability function (usually a single, key-parameter function), using motor overtest data, to validate model predictions with the actual test results. Similarly, a second requirement function is established and probilistically compared with the corresponding capability function for each failure mode to estimate the probability of failure of each failure mode under operational conditions. Using regression analysis, the projected values of the key parameters then are incorporated into the requirement and capability functions for each failure mode for future time intervals. The individual probabilities of failure vs time for each failure mode are calculated and then are combined to give the probability of motor failure vs time.

Introduction

THIS paper describes the statistical methods associated with the Long-Range Service Life Analysis (LRSLA) program. The LRSLA program was established to evaluate the effects of aging on performance and reliability of certain of the Minuteman solid-propellant rocket motors.

A basic concept of the LRSLA program is that detailed engineering analysis of a solid rocket motor will reveal the most probable failure modes for that motor (for more details of this selection process, see Ref. 1); that each hypothesized or actual failure can be described in terms of a quantitative motor requirement using a structural model; and that test failure will occur when the requirement exceeds the actual capability of the materials. For example, the motor requirement for the failure mode propellant-to-liner bond stress (to be withstood during pressurization of the motor) is established theoretically by a complex computerized structural model, defined as the requirement function, which is responsive to environments, time, and material properties. Given exact values of these parameters leading to a requirement, and exact values of the capability of the materials, the model, if valid, should predict failure or no failure for each set of conditions according to whether the requirements exceed the capability or vice versa. It is therefore possible to validate and calibrate the model using specified values of the parameters, imposing known conditions on the motor and observing whether failure occurs in actual test as predicted by the model. In practice, each test is run as an "overtest" in order to precipitate failure, thereby improving the efficiency of the validation process. The factors and levels establishing the overtest loads are determined using the computerized structural models. It is found that, generally, thermal or pressure loads can be used to simulate the effects of the aged operational conditions. The nature of the overtest is such as to take an unaged motor and apply the appropriate overtest loads until a failure occurs. This failure occurs under "nondestructive" conditions, i.e., conditions which allow, for example, the debonding to be observed, the motor to be dissected subsequently, and its properties analyzed in detail. This differs from the test situation in which a hot firing has taken place and only a limited amount of data is available.

The foregoing describes the deterministic approach to model validation whereas, in fact, the observed values of the parameters, which constitute the structural model, vary statistically and with time because of manufacturing and material property variation, including possible age degradation, as well as because of test measurement error and random variation; therefore, the model validation process must be performed probabilistically using the statistical methods described in detail later. Once the model has been validated, prediction of future probabilities of motor failure can be made using the model, based on predicted changes with time in the individual parameters.

This paper discusses the statistical techniques used to validate the model and to predict the motor's future failure probability. In particular, the approach to be discussed will show how the techniques of statistical design and analysis of variance can be used to greatly reduce the cost and effort required to validate the model, and to arrive at a simplified but valid linear expression of the parameters of the model for use in the probabilistic analysis. Also, it will be shown that the linear model, along with predictive techniques for future values of the parameters, is used for estimating the probability of motor failure as a function of age since it is the purpose of the statistical analysis to determine at what time in the future the probability of motor failure reaches an unacceptable level. The probability of motor failure is computed by mathematically combining the individual probabilities of failure of each critical failure mode.

The first step in the statistical approach is to regard the computerized structural model, i. e., the requirement function, as a statistical distribution function generated as the individual parameters "take on" their various statistical values. Analytically, there are several methods of generating the requirement distribution from the computerized structural model. The most obvious approach perhaps is to use Monte Carlo simulation. However, in this case it is expensive to make more than a limited number of runs of the model, and Monte Carlo simulation frequently requires several thousand runs. Instead, statistical design and analysis of variance techniques were used to obtain the requirement distribution. This is a new application of these techniques which have been developed independently and recently described in articles by Johnson, Maxwell, and Allred.[2,3] They significantly reduce

Presented as Paper 76-746 at the AIAA/SAE 12th Propulsion Conference, Palo Alto, Calif., July 26-29, 1976; submitted Aug. 9, 1976; revision received Jan. 10, 1977.

Index categories: Computer Technology; Reliability, Maintainability, and Logistics Support; Structural Durability

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the number of structural model computer runs that would be necessary using the Monte Carlo simulation approach, and they also have some additional advantages in model building. For example, the relative importance of the various parameters within the structural model can be established by standard analysis of variance F-tests. Also, the effects and standard deviations of each of the parameters can be estimated from the analysis of variance table. Further, the structural model can be approximated within specifiable limits by the previously mentioned, locally linear (requirement) function of the significant (or key) parameters. Since the parameters are approximately normally distributed, the linear equation then can be used to obtain the mean, the standard deviation, and, consequently, the distribution of the requirement function. This linear expression can be checked easily to determine where divergence from the computerized structural model takes place. If necessary, a second or third linear requirement expression can be generated for other conditions of interest, e.g., other overtest and/or operational conditions. The linear expression permits any parametric sensitivity analyses to be performed rapidly and the identification of the key parameters permits a significant reduction in the number of parameters which need to be tracked, since, by definition, any changes in nonsignificant parameters have minimal effect on the model and therefore on any probability of failure predictions.

In addition, since the linear expression is derived as a function of the key parameters it is necessary only to examine the behavior of the individual key parameters with time, in order to establish the behavior of the requirement distribution with time. Therefore, if the behavior of the capability distribution over time also is observed and estimated, the future probability of failure of an individual failure mode can be computed by comparing the requirement distribution with the capability distribution at periodic future times. The effects of age degradation and decreasing confidence can be seen in Fig. 1 as the requirement and capability distributions converge and overlap and increase the probability of failure.

The probabilities of individual failure modes over time subsequently are combined to obtain the probability of motor failure vs time. From this relationship the service life can be estimated as shown in Fig. 2 as that point in time where 90% confidence limit on the probability of motor failure intercepts a specified failure rate criterion line.

Methodology

The statistical methodology consists of four tasks: 1) establishing the linear requirement function and the capability function for each failure mode, 2) computing the probability of failure of individual failure modes by Requirement vs Capability analysis, 3) obtaining the regression statistics of

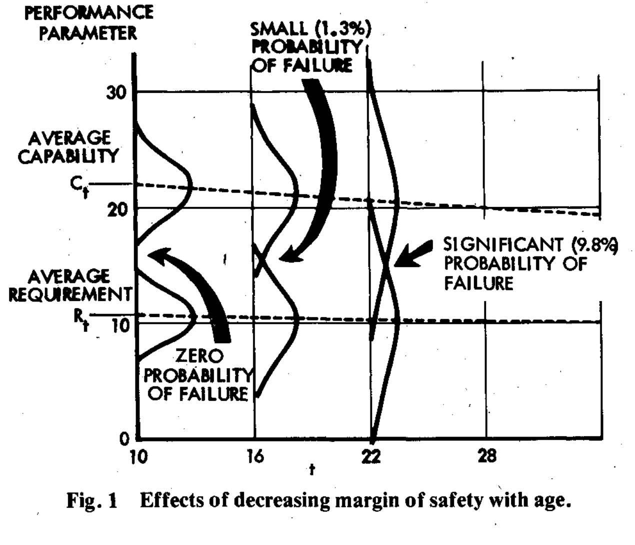

Fig. 1 Effects of decreasing margin of safety with age.

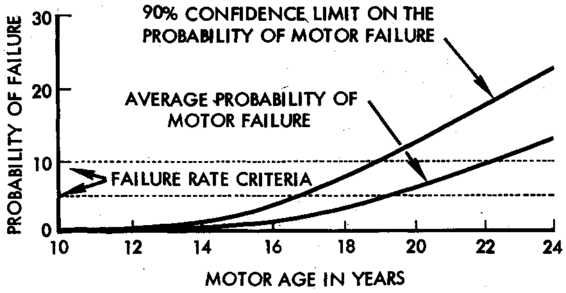

Fig. 2 Probability of failure vs age.

the key parameters over time, and 4) combining the probability of failure of individual failure modes over time to obtain the probability of motor failure vs time. The failure mode "debond initiation" for the Stage I Minuteman motor has been chosen to illustrate the computations associated with the aforementioned tasks.

Linear Requirement Function(s) and Capability Function(s)

Two linear requirement functions are required – one to approximate the structural model at overtest conditions for statistical model validation purposes, and the second to approximate the model at operational conditions to use in the probability of failure predictions.

Analysis of Overtest Conditions

The first step is to calculate the means and standard deviations from the overtest motor data of all the parameters which appear explicitly in the structural model and identify which of them, when changed by a Δ value, probably would affect the output of the model to a nontrivial extent. Also, any parameters which are known or conjectured to change with age also are noted. All such parameters are, at this point, considered potentially "key" on the basis of engineering judgment, and may include most if not all of the parameters of the structural model. The next steps, which are the selection and application of the statistical design and analysis of variance technique, result in the identification of, and probable reduction in, the number of parameters designated key.

The selection of the particular statistical design is based on 1) the number of potential key parameters, 2) the number of structural model computer runs that can be afforded, and 3) the availability of a suitable statistical design. The statistical designs used in this analysis were taken from Sec. 12-4 of Ref. 4 and are called fractional factorials. The theory and application of such designs is highly specialized and is beyond the scope of this paper; therefore, the reader is referred to the references for general reading on this subject. Two levels of each factor (parameter) are used in the selected design. Conventionally denoted as the upper(+) and lower(−) levels of each factor, the two levels are obtained simply as the average value of the parameter plus/minus a constant multiplied by the standard deviation of the parameter. The multiplying constant is taken to be 1 for overtest and 1.5 for operational conditions. The smaller range chosen for overtest conditions is to increase accuracy of the validation and calibration procedure when using the linear expression to predict the outcome of a particular motor for a particular overtest. The wider range is used for operational predictions because the spread of the data is greater and has to cover motor-to-motor variability, not within-motor variability.

The purpose of the first statistical design is to examine the behavior of the model in the region of interest, i.e., overtest average values of the parameters plus/minus one standard deviation. Table 1 illustrates the initial list of potential key parameters for the Stage I failure mode, and their upper and lower levels for subsequent incorporation into the selected statistical design shown in Table 2.

Table 1 Parameters and values in statistical runs for Stage I motor propellant/liner debond failure mode

CODE PARAMETER	NOMINAL	1 SIGMA VARIATION	LOWER VALUE	UPPER VALUE
A. Cylinder Thickness(inch)	0.1440	0.0144	0.1296*	0.1584*
B. Ins.Relaxation Mod(psi)	963.0	63.0	900	1026
C. Ins.Cof.Li.Exp.(in/in/deg)	102	2.65	99.35	104
D. Liner Relaxation Mod(psi)	193	35.6	157.4	228.6
E. Liner Cof.Li.Exp(in/in/deg)	106	3.56	102.44	109.56
F. Prop.Relaxation Mod(psi)	223	23.0	200	246
G. Prop.Cof.Li.Exp.(in/in/deg)	61.4	4.23	57.17	65.63
H. Test Temperature(degrees F)	20		18	22

Table 2 illustrates the statistical design (Plan 8, pp. 12-18 of Ref. 4) selected for examining the effects of eight factors (parameters) in 16 computer runs of the structural finite-element model. Plan 8 organizes the combinations of factors and their levels so that the effects of each of the eight factors can be estimated separately without being confounded with each other and/or interacting factors. It assumes that the effects of any interactions between factors will be small and can be used in the estimate of the residual variation. That these assumptions are valid can be seen by inspection of Table 3 and are discussed later in the analysis of the statistical design.

The output of a single computer run is shown in the last column of Table 2 labelled "Stress" in the row corresponding to the input set. For those parameters not varied in the computer runs (all but the eight parameters listed in Table 1, i.e., A through H), nominal values from the overtested motor are used. Table 3 is the analysis of variance associated with the statistical design. It should be noted that "Stress' represents the predicted bond strength requirement, in psi, as computed by the structural model. If the corresponding capability were less than this requirement, then, theoretically, in a nonprobabilistic sense, failure would occur.

The format of the Table 3 printout is in accordance with Plan 8, pp. 12-18 of Ref. 4. The design is a 1/16 fractional factorial design of eight factors (parameters) each at two levels. The run (treatment combinations 1-16) sequence is analyzed by the Yates Method to give the corresponding "Est Effect" shown in column 5 of the printout. The magnitude and significance of the effects are given, respectively, in columns 4 and 6 under the headings of "Ave Effect" and "Mean SQ." Factors F, G, and H have significant or very significant *F* ratios. The residual variance of 1.959 is used in the computation of the variance of the Requirement *R*

Table 3 Analysis of variance, overtest conditions

COMB	RUN	OPERATION	AVE EFFECT	EST EFFECT	MEAN SQ
1	33.460	631.920			
2	37.890	6.100	.762	A	2.3256
3	48.580	-8.660	-1.082	B	4.6872
4	40.320	3.760	.470	AB+CE+DG+FH	.8836
5	40.700	-5.420	-.678	C	1.8360
6	48.990	10.760	1.345	AC+BE+FG+DH	7.2361
7	31.960	-6.040	-.755	BC+AE+DF+GH	2.2801
8	33.200	-6.140	.767	E	2.3562
9	46.820	1.820	.227	D	.2070
10	41.820	-5.400	-.675	AD+EF+BG+CH	1.8225
11	31.690	5.400	.675	BD+AG+CF+EH	1.8225
12	38.140	43.140	5.392	G	116.3162
13	38.580	5.280	.660	CD+AH+BF+EG	1.7424
14	32.030	-15.860	-1.983	H	15.7212
15	41.170	78.020	9.753	F	330.4450
16	46.620	-5.040	-.630	AF+DE+CG+BH	1.5376
	631.920 = TOTAL			TOTAL =	541.2694

discussed later. The tests of significance are based on pooling all nonsignificant effects except the interaction factor (AC + BE + FG + DH) which has a marginally high mean square compared with the remainder of the nonsignificant mean squares.

Analyzing the Statistical Design

The results of the analysis of variance are as follows. Three factors were found to have a significant effect on the bond stress (requirement function): factor F, propellant modulus (E_R); factor G, propellant thermal coefficient of linear expansion (TCLE); and factor H, temperature. The most significant parameter is propellant modulus, with the propellant thermal coefficient of linear expansion being very significant also. The effects of all of the other parameters can be considered second order and omitted from the linear requirement function. Thus, the effects and relationship of the parameters can be expressed by the following equation. Requirement *R* is given by

$$R = 39.5 + (0.2120)(E_R - 223) + (0.6373)(\text{TCLE} \times 10^6 - 61.4) - (0.4957)(\text{temp} - 20) \quad \text{psi} \tag{1}$$

This equation is derived by dividing the respective average effects of the significant parameters, as shown in column 4 of Table 3, by the range of those parameters as inputs to the model. For example, over a range of 46 psi for propellant relaxation modulus the requirement changed 9.753 units or 0.2120 units per unit of change of propellant relaxation modulus. These changes occur about the nominal values of each significant input parameter shown in the Table 1, i.e.,

Table 2 Input and output values of matrix, overtest conditions

NO.	A	B	C	D	E	F	G	H	STRESS
RUN 1	0.1296	900	99.35E-6	157.4	102.44E-6	200	57.17E-6	123.0	33.46
RUN 2	0.1584	900	99.35E-6	157.4	109.56E-6	200	65.63E-6	119.0	37.89
RUN 3	0.1296	1026	99.35E-6	157.4	109.56E-6	246	65.63E-6	123.0	48.53
RUN 4	0.1584	1026	99.35E-6	157.4	102.44E-6	246	57.17E-6	119.0	40.32
RUN 5	0.1296	900	104.65E-6	157.4	109.56E-6	246	57.17E-6	119.0	40.70
RUN 6	0.1584	900	104.65E-6	157.4	102.44E-6	246	65.63E-6	123.0	48.99
RUN 7	0.1296	1026	104.65E-6	157.4	102.44E-6	200	65 63E-6	119.0	31.96
RUN 8	0.1584	1026	104.65E-6	157.4	109.56E-6	200	57.17E-6	123.0	33.20
RUN 9	0.1296	900	99.35E-6	228.6	102.44E-6	246	65.63E-6	119.0	46.82
RUN 10	0.1584	900	99.35E-6	228.6	109.56E-6	246	57.17E-6	123.0	41.82
RUN 11	0.1296	1026	99.35E-6	228.6	109.56E-6	200	57.17E-6	119.0	31.69
RUN 12	0.1584	1026	99.35E-6	228.6	102.44E-6	200	65.63E-6	123.0	38.14
RUN 13	0.1296	900	104.65E-6	228.6	109.56E-6	200	65.63E-6	123.0	38.58
RUN 14	0.1584	900	104.65E-6	228.6	102.44E-6	200	57.17E-6	119.0	32.03
RUN 15	0.1296	1026	104.65E-6	228.6	102.44E-6	246	57.17E-6	123.0	41.17
RUN 16	0.1584	1026	104.65E-6	228.6	109.56E-6	246	65.63E-6	119.0	46.62

for propellant relaxation modulus the nominal value is 223 psi. The constant $39.495 \approx 39.5$ is the average value of all 16 computer runs, i.e., 631.92/16. The identical equation also can be obtained directly by regression analysis.

Overtest Requirement and Capability Statistics

The standard deviation of the requirement distribution is obtained as the square root of the variance of linear expression Eq. (1). Although temperature is not random, Var (temp)→0; therefore the variance of the requirement distribution is given by

$$\text{Var}R = (.2120)^2[\text{SD}(E_R)]^2$$

$$+ (.6373)^2[\text{SD}(\text{TCLE}\times 10^6)]^2 + \sigma_0^2$$

where SD( ) is the abbreviation for the standard deviation of the parameter in the parentheses and σ_0^2 is the residual variation. Using data from Tables 1 and 3

$$\text{Var } R = 23.78 + 7.268 + 1.959 = 33.008 \text{ psi}^2$$

The standard deviation of R is equal to the square root of 33.008, i.e.

$$S_R = 5.745 \text{ psi}$$

The foregoing equations and values for the requirement distribution and its standard deviation hold for parameter values lying between the lower and upper values shown in Table 1. Therefore, for overtest motor S/N 12628 ($\text{TCLE} = 61.4 \times 10^6$ in./in./°F, $E_R = 223$ psi, temperature = 20°F), i.e., at overtest conditions, the mean value of the requirement distribution $\bar{R}$ is 39.5 psi with a standard deviation S_R of 5.74 psi.

Bond tensile stress is the parameter which defines the capability distribution. An analysis of variance was performed for each of the four test rates, at which the parameter was measured using dissected material, to determine the poolability of the data between the motors and areas within the motors. It was concluded from the analysis that the data were not poolable and therefore the specific data from the area of interest of the overtest motor (i.e., area I, motor S/N 12628) must be used as the nominal value. When computed for the time of interest, i.e., 72 days at 20°, the mean value for the capability distribution $\bar{C}$ is equal to 40.0 psi and the associated standard deviation, S_C, is 4.25 psi.

Analysis of Operational Conditions

The preceding statistics for the requirement and capability functions at overtest conditions are needed to validate statistically the structural model, as will be described presently. However, first, without repeating all of the details, the linear requirement and capability functions for operational conditions were derived in a manner similar to that used in the overtest analysis. Tables 4-6 contain the key parameter data for operational conditions, the input and output of the computer runs, and the analysis of variance of those runs, which lead to the linear requirement function, Eq. (2), at operational conditions.

Table 4 Nominal and range values for potential key parameters at operational conditions

CODE	PARAMETER	NOMINAL	1.0 SIGMA VARIATION	(+1.5 SIGMA) LOWER VALUE	UPPER VALUE
A	Ins.Relaxation Mod.(psi)	692.0	100.0	542	842
B	Prop.Relaxation Mod.(psi)	104.7	14.8	82.5	126.9
C	Prop.Cof.Li.Exp.(in/in/deg)	55.9E-6	3.65E-6	50.4E-6	61.4E-6
D	Test Temperature/Delta T-Deg	60		57	63

Table 5 Statistical plan and input/output matrix, operational conditions

RUN NO.	A	B	C	D	STRESS
1	542	82.5	50.4E-6	57	7.74
2	842	82.5	50.4E-6	63	6.81
3	542	126.9	50.4E-6	63	11.69
4	842	126.9	50.4E-6	57	12.02
5	542	82.5	61.4E-6	63	8.98
6	842	82.5	61.4E-6	57	9.20
7	542	126.9	61.4E-6	57	15.67
8	842	126.9	61.4E-6	63	13.95

The format of the analysis of variance table is in accordance with Plan 2, pp. 12-16 of Ref. 1. The design is a ½ fractional factorial design of four factors (parameters) each at two levels. The run (treatment combination 1 through 8) sequence is analyzed by the Yates Method to give the corresponding "Est Effects" shown in column 5. The F-test values are based on an error variance of 0.34.

Operational Requirement and Capability Statistics

Requirement R is given by

$$R = 10.76 + .1160(E_R - 104.7) + .2168\ (\text{TCLE}\times 10^6 - 55.9)$$

$$-.1333(\text{temp} - 60) \text{ psi} \tag{2}$$

Therefore $\bar{R} = 10.76$ is the mean operational requirement when E_R, TCLE, and temperature take on their nominal values. $S_R = 1.98$ psi is its associated standard derivtion. The mean capability $\bar{C} = 22.5$ psi, and its associated standard deviation, $S_C = 3.24$, are obtained from data from dissected motors. These statistics will be used subsequently to compute the probability of failure due to debond initiation, using Requirement vs Capability analysis.

Requirement vs Capability Analysis

The outcome of any test can be predicted probabilistically by comparing the requirement distribution with the capability distribution using the following equation. The average probability of failure

$$\bar{P}_F = 1 - \Phi[(\bar{C} - \bar{R})/(S_C^2 + S_R^2)^{½}] \tag{3}$$

where $\Phi[\ ]$ is the cumulative normal distribution function and can be evaluated using Appendix Table I of Ref. 5.

The 90% confidence limit on the probability of failure is given by

$$P_{F,90} = \bar{P}_F + t_{0.1,n-1}(\text{Var } P)^{½} \tag{4}$$

where

$$\text{Var } P = \frac{1}{2\pi}\exp - \left(\frac{\bar{C}-\bar{R}}{(S_C^2+S_R^2)^{½}}\right)^2 \left[\frac{1}{(S_C^2+S_R^2)}\left(\frac{S_C^2}{n_C} + \frac{S_R^2}{n_R}\right)\right.$$

$$\left. + \frac{(\bar{C}-\bar{R})^2}{(S_C^2+S_R^2)^3}\left(\frac{S_C^2}{2n_C} + \frac{S_R^2}{2n_R}\right)\right] \tag{4a}$$

$t_{0.1,n-1}$ is "Student's t" evaluated at the 0.1 percentage point with $n-1$ degrees of freedom (Appendix Table III of Ref. 5) and n_C, n_R are the respective numbers of key parameter values upon which the capability and requirement statistics are based. The expression for Var P, i.e., Eq. (4a), is derived in the Appendix.

Generally in the foregoing equations, $n_C = n_R = n$; however, if $n_R \neq n_C$, then n should be chosen as the smaller of n_C, n_R.

Table 6 Analysis of variance table, operational conditions

COMB	RUN	OPERATION	AVE EFFECT	EST EFFECT	MEAN SQ	SIGNIF F-TEST
1	7.740	86.060				
2	6.810	-2.100	-.525	A	.5513	1.62
3	11.690	20.600	5.150	B	53.0450	156.01XX
4	12.020	-.680	-.170	AB+CD	.0578	-
5	8.980	9.540	2.385	C	11.3765	33.46XX
6	9.200	-.900	-.225	AC+BD	.1013	-
7	15.670	2.280	.570	BC+AD	.6498	-
8	13.950	-3.200	-.800	D	1.2800	3.76
	86.060 = TOTAL				TOTAL=67.0615	

The computation gives the average probabilities of failure and 90% confidence limits for the failure mode for the age of the material tested.

Statistical Model Validation

Using the overtest statistics for $\overline{R}$, $\overline{C}$, S_R, and S_C and Eq. (3) we have

$$\overline{P_F} = 1 - \Phi[(40.0-39.5)/(4.25^2+5.74^2)^{1/2}]$$

which is computed to equal 47%. This probability can be compared with the theoretical probability of 50% wherein the requirement is exactly and instantaneously equal to the capability at the time of failure. Therefore, within the limits of test data variation, it is concluded that this calculated probability is sufficiently close to the theoretical value that the model can be considered statistically valid for overtest motor S/N 12628. Similarly, for other overtest motors, their key parameter statistics are inserted into the linear requirement expression (1) to obtain the corresponding values of R which are compared probabilistically with their corresponding capability statistics using Eq. (3). In each case, at the conditions of instantaneous failure, the computed probability of failure should approach 50%. When this occurs, the model is considered to be validated statistically and ready for general use, including its application at operational conditions.

Operational Probability of Failure

The next use of Eqs. (3) and (4) is the computation of operational probability of failure for each failure mode. The requirement and capability functions and statistics, which were derived for operational conditions, now are used. Substituting the operational statistics from the previous section into Eq. (3) gives

$$\overline{P_F} = 1 - \Phi[(22.50-10.76)/(3.24^2+1.98^2)^{1/2}]$$
$$= 0.000983 < .01\%$$

The 90% confidence limit on the probability of failure is calculated, using Eq. (4), to be 0.94%. The 90% confidence limit is an important statistic because it is the measure against which the service life criterion is applied. For example, in Fig. 2 the service life is defined as the age at which the 90% confidence limit on the probability of failure intersects the selected failure rate criteria line. Requirement vs capability analysis also is used to compute future probabilities of failure. By definition, only changes in the values of key parameters affect the requirement function; therefore it is necessary to examine only the changes in the key parameters with time to determine how the requirement and capability functions are changed with time.

Age Regression Statistics for Requirement and Capability Functions

In order to compute future values of the requirement and capability functions it is first necessary to obtain age regression equations for each of the key parameters of the failure modes; in particular the estimates of the average and standard deviation values of the key parameters at any future point in time.

Consider a series of observations y_i of a key parameter taken at time, t_i, then using linear† regression analysis (paragraph 9.7.1, Ref. 6) the future average value $\bar{y}_t$ of the key parameter and its associated standard deviation S_t at time t are given by

$$\bar{y}_t = \bar{y} + b(t-\bar{t}) \quad (5)$$

$$S_t = S_y\{1+(1/n)+[(t-\bar{t})^2/\Sigma(t_i-\bar{t})^2]\}^{1/2} \quad (6)$$

where

$$b = [\Sigma(t_i-\bar{t})(y_i-\bar{y})]/\Sigma(t_i-\bar{t})^2$$

i.e., b is the slope of the regression line, and

$$S_y = \left\{\frac{1}{(n-2)}\left[\Sigma(y_i-\bar{y})^2 - \frac{[\Sigma(t_i-\bar{t})(y_i-\bar{y})]^2}{\Sigma(t_i-\bar{t})^2}\right]\right\}^{1/2}$$

i.e., S_y is the residual standard deviation.

After computing the preceding statistics the slope b is tested, using the t-test at the 5% significance level, to determine whether it is significantly different from zero. If b is significantly different from zero, then the calculated value is used in Eq. (5); however, if b is not significantly different from zero, then b is assumed to be zero in Eq. (5). This is done to eliminate the effects of random variation influencing the calculated value of b when given the null hypothesis, $b=0$, and therefore preventing the future key parameter values from fluctuating unnecessarily each time new data are added.

†It is assumed that linear regression best estimates the degradation effects of aging and, therefore, it is used throughout this program. However, if data trends are curvilinear as determined by observation and statistical test, the basic approach to computing the probability of failure vs time remains the same. Equation (5) would be modified to include polynomial or even nonlinear terms; however, the mathematical equivalent to Eq. (6) for curvilinear regression has not yet been derived in the literature. An approximate way of proceeding might be to use the curvilinear equation equivalent to Eq. (5) and assume a constant variance with time which would give a nonconservative answer, or use the curvilinear equation equivalent to (5) and Eq. (6) as it stands, but substitute the standard deviation of the variable at a single time point (i.e., the age of the oldest motor) for S_y in Eq. (5).

If the null hypothesis does not hold, then the formula for b gives the best estimate of what the true value of b is, and Eq. (5) is used to calculate future values of $\bar{y}_t$ using this estimate. Whether b is zero or not, the value of each key parameter $\bar{y}_t$ obtained from Eq. (5) represents chemical aging only, and, therefore, $\bar{y}_t$ also must be adjusted using time-temperature superposition, from the master curves, to take into account the mechanical response with time due to the viscoelastic properties (mechanical aging). Thus, the value $\bar{y}_t$ as subsequently computed for E_R and σ_m includes both mechanical and chemical degradation and is referred to as the "future value" of the key parameters in subsequent equations.

First, the regression formulas (5) and (6) are applied to the age data given in Table 7 of each of the following key parameters: E_R (propellant relaxation modulus), TCLE (propellant thermal coefficient of linear expansion), and σ_m, (propellant/liner/insulation bond tensile strength). Statistical analysis showed that the slopes of all three regression lines are not significantly different from zero, and are therefore taken to be equal to zero. Thus, from a chemical aging viewpoint, the values of the three parameters are equal to the constant term in Eq. (5) and need be adjusted only for the conditions of interest, 60°F, and mechanical aging. Since TCLE is not subject to mechanical aging, the adjustments are necessary only for E_R and bond tensile strength. The adjusted values of E_R and bondline tensile strength are obtained from their master curves and are given in Table 8.

Equation (2) allows E_R to be combined with the TCLE and temperature parameters to give the future values of the mean requirement. In Eq. (2) the silo temperature is taken to be 60°F and the average value of the operational motors' TCLE is equal to 56.2×10^{-6} in./in./deg.

Since the bondline tensile strength is also the capability parameter, the average requirement and capability values are given in Table 9. The next step is to calculate the "future" standard deviations of each of the key parameters. Substituting the regression statistics in Eq. (6) and calculating S_t vs time for each of the three parameters gives the values in Table 10. Next we combine SD (E_R) and SD (TCLE) using Eq. (2), i.e., Var $R_t = .34005 + (.11599)^2$ [Future SD $(E_R)]^2 + (.21682)^2$ [Future SD (TCLE $\times 10^6$) to give Table 11. We now have R_t, S_{R_t}, $\bar{C}_t$, and $S_{C_t} \equiv$ SD (bond tensile strength) for values of $t = 10, 12, \ldots$ years (and also $n_R = 10$, $n_C = 16$, from Table 7), enabling the average probability of failure and the associated confidence limits to be computed using Eqs. (3) and (4). The results of the computation are shown in Table 12.

Table 7 Stage I key parameter values at test conditions

Motor	Age in Months	E_R 180 @ 1.22	TCLE x10^6	Tensile 180°@.02
STM-021	140	162	52.04	20.7
12638	114	196	54.9	29.3
12628	118	189	61.38	35.3
12447	120	160	53.22	32.0
12660	112	177	52.82	28.3
12380	123	172	60.95	34.7
12705	114	157	54.4	29.3
12750	113	178	57.52	32.0
13075	93	168	57.9	31.3
13327	74	161	56.65	27.7
12705	121	-	-	33.3
12380	133	-	-	33.3
12447	130	-	-	32.0
12705	127	-	-	29.7
12380	139	-	-	29.7
12447	136.5	-	-	31.0

Table 8 Future values of key requirement parameters

Age in Yrs.	10	12	14	16	18	20	22	24
E_R	103.2	101.1	99.8	99.6	99.5	99.4	99.1	98.6
Bond Tensile	22.2	22.0	21.7	21.4	21.2	21.0	20.7	20.4

Table 9 Future values of mean requirement and capability functions

Age in Yrs.	10	12	14	16	18	20	22	24
$\bar{R}_t$	10.6	10.4	10.2	10.2	10.2	10.2	10.2	10.1
$\bar{C}_t$	22.2	22.0	21.7	21.4	21.2	21.0	20.7	20.4

Table 10 Future standard deviations of key parameters

Age in Yrs.	10	12	14	16	18	20	22	24
SD(E_R)	14.558	16.611	20.424	25.209	30.513	36.108	41.877	47.758
SD(TCLE)	3.599	4.107	5.050	6.233	7.544	8.928	10.354	11.808
SD(Bond TS)	2.603	2.769	3.199	3.805	4.516	5.290	6.103	6.942

Table 11 Future standard deviation of the requirement function

Age in Yrs.	10	12	14	16	18	20	22	24
$(\mathrm{Var}\ R_t)^{1/2} \equiv S_{R_t}$	1.95	2.20	2.67	3.27	3.94	4.65	5.38	6.13

Table 12 Future probabilities of failure, debond initiation

t	$\bar{R}_t$	$\bar{C}_t$	S_{R_t}	S_{C_t}	$\bar{P}_F$	$P_{F,90\%}$
10	10.6	22.2	1.95	2.60	.00018	.00184
12	10.4	22.0	2.20	2.77	.00052	.00371
14	10.2	21.7	2.67	3.20	.00289	.01154
16	10.2	21.4	3.27	3.81	.01285	.03613
18	10.2	21.2	3.94	4.52	.03329	.07535
20	10.2	21.0	4.65	5.29	.06259	.12534
22	10.2	20.7	5.38	6.10	.09836	.17730
24	10.1	20.4	6.13	6.94	.13299	.22695

Combining Individual Failure Mode Probabilities of Failure

The probability of failure computations just described were performed for all four potential critical failure modes of Stage I, i.e., debond initiation, debond propagation, crack initiation, and crack propagation. Therefore, in order to obtain the motor probability of failure it is necessary to combine these individual failure mode probabilities as follows.

If the occurrence of either of the failure modes, propellant cracking or propellant/liner debonding, causes mission failure, then the motor is, by definition, a serial system. This is expressed mathematically by

$$P(M_f) = 1 - [1 - P(D_f)][1 - P(C_f)] \qquad (7)$$

where $P(M_f)$ is the probability of motor failure, $P(D_f)$ is the probability of motor failure due to debonding, and $P(C_f)$ is the probability of motor failure due to cracking. Now

$$P(D_f) = P(D_i) \times P(D_p | D_i) \qquad (8)$$

and

$$P(C_f) = (P(C_i) \times P(C_p | C_i) \qquad (9)$$

where $P(D_i)$ and $P(C_i)$ are the probabilities of debond initiation and crack initiation, respectively, and $P(D_p | D_i)$ and $P(C_p | C_i)$ are, respectively, the conditional probabilities of debond propagation, given debond initiation, and crack propagation, given crack initiation.

Analysis shows that propagation will occur once debond initiation occurs; conversely, analysis shows that crack propagation will not occur beyond the 3.5 in. maximum allowable depth to cause mission failure. In terms of the preceding mathematical equations, the foregoing statements imply that $P(D_p | D_i) = 1$, and $P(C_p | C_i) = 0$. Substituting in Eqs. (7-9) gives

$$P(M_F) = 1 - [1 - P(D_i) \times P(D_p | D_i)] [1 - P(C_i) \times P(C_p | C_i)]$$
$$= 1 - [1 - P(D_i)] [1 - 0] = P(D_i)$$

i.e., the probability of motor failure is identical to the probability of debond initiation; however, we have the calculated probabilities of debond initiation $P(D_i)$ in the previous section as $\bar{P}_F$ and $P_{F,.90}$; thus, the probability of motor failure vs time is obtained directly from the last two columns in Table 12.‡ These values were plotted in Fig. 2 as the probability of motor failure vs time.

Conclusion

The statistical methods used in the LRSLA program for predicting service life have been cost effective in being able to examine the behavior and projected behavior of the critical failure modes using a very limited number of the structural model computer runs. Methods for statistically validating the models with overtest results and subsequently computing the probability of motor failure vs time have been developed which are based on identifying a limited number of key parameters in each model. This not only facilitates sensitivity analysis of motor behavior but also reduces the cost of surveillance program activities by reducing the number of parameters which need to be monitored to determine future trends of motor ageout.

Appendix: Analytical Method for Obtaining Confidence Limits

The equations contained in this Appendix are based on the method of Chaps. 8 and 9 of Ref. 7, and in particular Secs. 8.4 and 9.3.3.

The variance of function $F(x_i)$, where x_i, $i = 1, \ldots, n$, are n independent random variables, is given by

$$\text{Var}(F) = \sum_{i=1}^{n} \left(\frac{\partial F}{\partial x_i}\right)^2 \text{Var}\, x_i$$

‡In general the problem of combining individual confidence limits to obtain system confidence limits, when there are two or more subsystems or failure modes, is a difficult one and various methods for obtaining the system confidence limits are discussed in Sec. 10.4 of Ref. 8. Thus, although Eq. (7) is used to combine average probabilities of failure, i.e., $\bar{P}_F$, it would *not* be used to combine confidence limits of two or more failure probabilities. The problem is not of immediate concern here; however, it is possible that it may occur at some future time if two or more failure modes per stage become critical, in which case the recommended method of computing system confidence limits is described in Sec. 9.2.3 of Ref. 7.

because when x_i are independent the covariances between x_i are all zero. Applying this to Eq. 3 we have

$$\text{Var}\, P = \left(\frac{\partial \Phi}{\partial \bar{C}}\right)^2 \text{Var}\bar{C} + \left(\frac{\partial \Phi}{\partial \bar{C}}\right)^2 \text{Var}\, \bar{R} + \left(\frac{\partial \Phi}{\partial S_C}\right)^2 \text{Var}\, S_C + \left(\frac{\partial \Phi}{\partial S_R}\right)^2 \text{Var}\, S_R \qquad (A1)$$

where

$$\frac{\partial \Phi}{\partial \bar{C}} = \frac{1}{(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\, \frac{(\bar{C}-\bar{R})^2}{(S_C^2+S_R^2)}\right] \frac{1}{(S_C^2+S_R^2)^{1/2}}$$

$$\frac{\partial \Phi}{\partial \bar{R}} = \frac{1}{(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\, \frac{(\bar{C}-\bar{R})^2}{S_C^2+S_R^2}\right] \frac{(-1)}{(S_C^2+S_R^2)^{1/2}}$$

$$\frac{\partial \Phi}{\partial S_C} = \phi\, \frac{(\bar{C}-\bar{R})(-S_C)}{(S_C^2+S_R^2)^{3/2}}$$

$$\frac{\partial \Phi}{\partial S_R} = \phi\, \frac{(\bar{C}-\bar{R})(-S_R)}{(S_C^2+S_R^2)^{3/2}}$$

where
$$\phi = \frac{1}{(2\pi)^{1/2}} \exp - \frac{1}{2}\left[\frac{\bar{C}-\bar{R})^2}{(S_C^2+S_R^2)}\right]$$

It is well known (see p. 194 of Ref. 4) that Var $C = S_C^2/n_C$, Var $R = S_R^2/n_R$, and Var $S_C = S_C^2/2n_c$, Var $S_R = S_R^2/2n_R$. Substituting in Eq. (A1) for the partial differentials and the variances, we get

$$\text{Var}\, P = (\phi)^2 \left\{ \frac{1}{(S_C^2+S_R^2)} \left(\frac{S_C^2}{n_c} + \frac{S_R^2}{n_R}\right) + \frac{(\bar{C}-\bar{R})^2}{(S_C^2+S_R^2)^3} \left(S_C^2 \frac{S_C^2}{2n_C} + S_R^2 \frac{S_R^2}{2n_R}\right)\right\}$$
$$= \frac{1}{2} \exp - \left(\frac{\bar{C}-\bar{R}}{(S_C^2+S_R^2)^{1/2}}\right)^2 \left\{ \frac{1}{(S_C^2+S_R^2)} \left(\frac{S_C^2}{n_C} + \frac{S_R^2}{n_R}\right) + \frac{(\bar{C}-\bar{R})^2}{(S_C^2+S_R^2)^3} \left(\frac{S_C^4}{2n_C} + \frac{S_R^4}{2n_R}\right)\right\}$$

where n_C and n_R are the sample sizes used to calculate the capability and requirement statistics, respectively. Thus, having P and the variance of P, confidence limits are obtained by assuming P is normally distributed and using the t-distribution in the usual manner.

Acknowledgment

This work was supported by USAF Contract No. F42600-76-C-0428; E.F. Lund, LRSLA Program Manager, Hill Air Force Base.

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